Utilitarian preferences with multiple priors*

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Abstract

This paper proposes a method for aggregating individual preferences in the context of uncertainty. Individuals are assumed to abide by Savage's model of Subjective Expected Utility, in which everyone has his/her own utility and subjective probability. Disagreement on probabilities among individuals gives rise to uncertainty at the societal level, and thus society may entertain a set of probabilities rather than only one. We assume that social preference admits a Maxmin Expected Utility representation. In this context, two Pareto-type conditions are shown to be equivalent to social utility being a weighted average of individual utilities and the social set of priors containing only weighted averages of individual priors. Thus, society respects consensus among individuals' beliefs and does not add ambiguity beyond disagreement on beliefs. We also deal with the case in which society does not rule out any individual belief.

Keywords: Preference aggregation, Decision under uncertainty, Multiple priors
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1 Introduction

Consider a government agency that is assumed to be benevolent and respectful of the preferences and beliefs of the constituents it serves. This government agency is required to make a policy decision. Constituents' goals, values, opinions, and predictions are not identical and the agency serving them is required to aggregate these tastes and beliefs. What would be an appropriate way of doing so?

This paper is based on the premise that in the absence of agreed-upon probabilities among members of society social planners face ambiguity. Assuming that planners have no authoritative external source of information, they use individuals' beliefs as a basis for forming a social belief. However, the very fact that beliefs differ raises doubt: is there a single, "correct" belief, and is it knowable? We suggest that in situations where social planners take influential and perhaps irreversible decisions, there is room for caution. In such situations social planners may be reluctant to commit to a single probability, preferring instead to implement a decision rule that accommodates ambiguity. Allowance for ambiguity does not preclude social planners from adopting a single probability as their belief, but the model presented in this paper provides them with the option to set policy that is sensitive to disagreement over beliefs.

The aforementioned rationale accords with the widely accepted precautionary principle which states that when an action or policy may cause harm to the public or to the environment the burden of proof that it is not in fact harmful falls on those taking the action. This principle was first endorsed internationally in the 1982 United Nations World Charter of Nature and has been incorporated into many other environmental treaties, such as the 1987 Montreal Protocol, the 1992 Rio Declaration, and the 1997 Kyoto Protocol. The principle has subsequently been implemented in many international and domestic regulations of genetically engineered products, cell-phone radiation, food safety, extinction of species, global warming, and so forth. While there is no single accepted interpretation of the precautionary principle – and its most extreme formulation requires absolute proof of safety before allowing an action to be taken – even in weaker interpretations the need for adopting a cautious approach in the face of uncertainty is expressed.

In this paper we propose a social welfare function that aggregates individual preferences when alternatives are uncertain. Uncertainty is modeled by an abstract set of states of nature as formulated in Savage's [34] theory of Subjective Expected Utility (SEU) according to which, individual preference admits an expected utility representation, whereby individual utilities over outcomes and individual prior probabilities over events both are subjective.

A common assumption in models of our type is that society conforms to the same decision rule as do individuals. In our context this means that the social planner adopts a single prior probability over events. However the mere fact that individuals hold conflicting evaluations suggests that no prescribed prior probability is obvious. Thus ambiguity arises at a societal level even when none exists on an individual level. By allowing social planners to entertain multiple priors, the decision rule is our model is able to accommodate and respond to this ambiguity. In as much as social planners are responsible for the well-being of all members of society and in line with the precautionary principle a cautious approach toward ambiguity is adopted. The social planner in our model evaluates each alternative by its minimal expected utility with respect to a set of priors as specified in Gilboa and Schmeidler's [21] Maxmin Expected Utility (MEU) decision rule.

A standard assumption binding social planning to the preferences of individual members of society is the Pareto principle. However it has long been recognized that the Pareto principle is problematic when beliefs as well as tastes differ across individuals. In the context of risk, where only tastes differ, Harsanyi [22] shows that under an expected utility hypothesis the standard Pareto principle is tantamount to social utility being a weighted sum of individual utilities. However, Mongin [26] following the example of Hylland and Zeckhauser [23] proves that it may be impossible to aggregate SEU individual preferences into a social preference that is consistent with both SEU and the standard Pareto condition.[1] Gilboa, Samet, and Schmeidler [19]

[1] Mongin's result relates to ideas found in Raiffa [33].

and Mongin [27], argue that the Pareto Principle might not even be very compelling in this context, as unanimous preferences may result from the cancelling out of individual disagreements on tastes and beliefs. In light of these discrepancies, this paper employs only those versions of the Pareto principle which guarantee expression of genuine agreement: one in which all individuals agree on probabilities, and the other in which they all agree on utilities. Both conditions have been introduced by Gilboa, Samet, and Schmeidler ([19]).

Lottery Pareto, our first Pareto criterion, states that for two acts that involve events whose probabilities are agreed upon by all, consensus on the ranking of these acts will have to be respected by society. For example, a firm employing ten workers is forced to slash its labor budget by 10%. Two possibilities are considered: reducing the salary of all employees by 10% (and correspondingly cutting down their work load) or laying-off one employee who is to be chosen at random. The distribution of outcomes is well understood by the employees and, being risk averse, all prefer the first alternative to the second. This consensual preference fits Harsanyi's model, as it involves commonly-shared probabilities. The firm, not having a preference for either option, fulfills the consensual wish of its employees, therein behaving as entailed in Lottery Pareto.

An analogous condition, Likelihood Pareto, compels the social planner to accept any unanimous preference concerning acts that are contingent upon the same pair of identically-ranked outcomes. To illustrate this, consider a sports team that is about to play its final game in an important championship. The team's star player has just been injured and a decision needs to be made whether to allow her to continue in the game or to put in a substitute player. This decision is to be taken by the coach, upon consultation with the captain and the manager. Given that there is no conflict of interest among individuals (coach, captain, manager) – all prefer winning to losing – differences in preferences are only due to differences in probabilities. If all prefer one option, say, the substitute player, it stands to reason that this be the coach's decision, as this consensual preference indicates that they all agree that the probability of winning with the substitute player is higher than that of winning with the injured star. [2]

Our main theorem shows that under the assumptions of SEU-maximizing individuals and an MEU-maximizing society, the two Pareto-type conditions are equivalent to the postulate that society's tastes are utilitarian, and society's belief consists of a set of prior probabilities that are weighted averages of individual priors. Thus individual tastes are aggregated as in Harsanyi's theorem and each social prior is a "utilitarian" aggregation of individual priors. Such an aggregation has been suggested by Gilboa, Samet, and Schmeidler [19] for the case of an SEU social planner whose set of social priors is a singleton. Our theorem generalizes their result in allowing society to express aversion to ambiguity in response to disagreement over beliefs.

An implication of this theorem is that whenever all individual priors assign the same probability to an event, then so do all priors of the social planner. Consequently, events obtaining the same probability assessment for all individuals are perceived as unambiguous by the social planner. Ambiguity only emerges when disagreement on probabilities among members of society prevails. In other words, under our assumptions the Pareto criteria entail that the only source of ambiguity can be conflicting probabilistic assessments on the part of individuals. Furthermore, the fact that social priors are weighted averages of individual probabilities is a reflection of the principle that social belief is based on, and composed exclusively of, individual beliefs.

The main theorem set forth in this paper does not require the set of priors representing social belief to contain all individual priors. The social planner is granted discretion in choosing the set of weights over individual beliefs, thus bounding their relative weights or even ignoring some of them altogether. This use of a pared down subset is appropriate in situations where only some of the individuals are considered to be knowledgeable in the question at hand. Nonetheless, there is also interest in the

[2] It should be noted that our interpretations of the Pareto criteria as well as the motivation for the choice of a social decision rule are based upon utilities and subjective probabilities in Savage's theory being meaningful representations of individual tastes and beliefs. This issue is discussed further in Subsection 3.1.

case where society views all individual priors as valuable and therefore includes them all in its set of priors. In a complementary result these cases are characterized by supplementing the previous assumptions with a strong notion of ambiguity aversion on behalf of the social planner.

1.1 More on the related literature

There is considerable literature on simultaneous aggregation of utilities and beliefs. In a generalization of the impossibility results of Mongin ([26] and [28]), Chambers and Hayashi [7] show that impossibility extends to a broader class of preferences, pointing to axioms in Savage's framework that are incompatible with the standard Pareto condition.

Broadening the scope of Mongin's impossibility result, Gajdos, Tallon, and Vergnaud [15] prove that for a wide class of preferences the standard Pareto condition is incompatible with aggregation of individual preferences into a social preference in cases where non-neutral attitudes towards ambiguity prevail. Our paper partially addresses this impossibility in the special case of SEU agents and an MEU society. More direct responses to [15] have been proposed by Danan, Gajdos, Hill, and Tallon [9] and Qu [32] that allow individuals as well as society to express non-neutral attitudes toward ambiguity. These two models, formulated in an Anscombe-Aumann setup ([3]), yield possibility results based upon Pareto conditions that are similar to those employed in the present work.[3]

As opposed to the papers just mentioned in which individuals and society are assumed to abide by the same decision rule, a distinction between decision rules applying to individuals and those applying to society is permissible in our thesis. This complies with Diamond's [12] claim that it may be normatively inappropriate to apply the same decision rule both to society and to individuals. Danan, Gajdos, and Tallon's [10] recent investigation of Paretian aggregation under an assumption of incomplete expected utility preferences for both individuals and society discusses, in

[3] Billot and Vergopoulos [4] in an alternative approach obtain a utilitarian social welfare function on an extended state space, imposing a standard Pareto condition.

the context of risk, the special case wherein individual preferences may be complete while those of society are incomplete. Society's preferences are represented by a unanimity rule over a set of convex combinations of individual utilities. This set of convex combinations is analogous to the set of priors composed of convex combinations of individual priors that is used in our model.

An incomplete social welfare criterion has also been provided by Brunnermeier, Simsek, and Xiong [6] for uncertain alternatives. According to their criterion, social planners evaluate outcomes via a utilitarian aggregate of individual tastes. An alternative is said to be superior to another if the expected social utility of the former is higher than that of the latter for every individual prior.[4] Under either of our two theorems in our model whenever such domination holds, the social planner follows the same ranking of alternatives. The notable conceptual difference between Brunnermeier, Simsek, and Xiong [6] and our paper is that while we axiomatically derive a complete social welfare function, they study the properties of their efficiency criteria, demonstrating their implications in various economic setups. Furthermore, in Brunnermeier, Simsek, and Xiong [6] heterogenous priors of agents are the result of behavioral biases that distort beliefs, these biases being their justification for weakening the standard Pareto criterion. By contrast, in our model standard Pareto is weakened so as to avoid cases where agreement is based on conflict.

An issue related to aggregation of individual preferences is that of a decision maker who consults with experts, who are assumed to share the decision maker's tastes but differ with respect to assessments of likelihoods. The aggregation of beliefs in our model resembles the aggregation of expert beliefs in Cres, Gilboa, and Vielle [8]: multiple weights are assigned to individual beliefs (be they those of experts or those of members of society) and the minimum utility over these weights is used for evaluating alternatives. However, while Cres, Gilboa, and Vielle [8] axiomatize aggregation of MEU beliefs, our paper axiomatizes aggregation of SEU beliefs. Another related decision rule, in the form of a minimum over more general opinions is Nascimento's

[4]These authors present another criterion of Pareto efficiency that is less related to the present work.

[30] allowance for expert opinions conveying different perceptions of ambiguity and different attitudes toward it.

Mongin and Pivato [29] take another approach to aggregating individual preferences. They investigate ex ante and ex post Pareto conditions in a multi-dimensional array framework wherein uncertainty is modeled by a two-component state: one corresponding to a subjective source of uncertainty and the other to an objective source. The utilitarian representation of society that they derive agrees with individual priors with regard to the objective state space, but makes no statement on the relationship between social and individual beliefs in the subjective dimension of the state space.

In light of the financial crisis of 2007/2008, several recent papers claim that the Pareto principle may be undesirable in connection with unregulated trade (see for instance Weyl [37]; Posner and Weyl [31]; Blume, Easley, Sargent, and Tsyrennikov [5]; and Brunnermeier, Simsek, and Xiong [6]). These papers claim that the standard Pareto criterion may not be appropriate in situations where agents hold heterogeneous beliefs: agents whose beliefs are close to the truth may prosper, while those holding 'wrong' beliefs may end up making poor decisions.

In the search for an appropriate Pareto criterion in uncertain environments Gilboa, Samuelson, and Schmeidler [20] present a model of a regulator whose objective is to interfere as little as possible with voluntary trade. They suggest that this regulator act in accordance with a Pareto criterion called no-betting Pareto, which requires that, on top of unanimity of personal preferences, there be a single probability that can rationalize the preferences of all SEU agents. This probability measure need not have any connection to the probabilities of individuals nor to that of the regulator. This is sensible when the sole objective of the regulator is to ban actions that are essentially betting. However, it is less satisfactory when making social decisions. In particular, there is no reason to expect that individual welfare increases when alternatives are evaluated using a probability measure that is unrelated to any individual's conception of reality. A related Pareto concept, which weakens standard Pareto but generalizes both Lottery Pareto and Likelihood Pareto, can be found in Gayer, Gilboa, Samuelson, and Schmeidler [16].

1.2 Outline of the paper

The next section contains the setup and the basic assumptions of the model. Section 3 presents the Pareto criteria and our two aggregation theorems. A discussion of identification of tastes and beliefs is given in Subsection 3.1. Finally, the proofs appear in Section 4.

2 Setup and basic assumptions

Let $N = \{1, \dots, n\}$ be the set of individuals whose preferences are to be aggregated. Subjective uncertainty is represented by a nonempty set S of *states of nature* that is endowed with a sigma algebra Σ of *events*. The set of possible *outcomes* is X and the set of (simple) *acts* is $\mathcal{F} = \{f \in X^S \mid f$ obtains finitely many values and is measurable w.r.t. $\Sigma\}$. For acts f and g and event E, fEg stands for the act which assigns the outcome $f(s)$ to $s \in E$ and $g(s)$ otherwise. The preferences of individual i are modeled by a binary relation, $\succsim^i$ over $\mathcal{F}$, with $\succ^i$ and $\sim^i$ being its asymmetric and symmetric components, respectively. Social preference is modeled by another binary relation over acts, $\succsim^0$, with asymmetric and symmetric components $\succ^0$ and $\sim^0$.

Throughout the paper it is assumed that individual preferences satisfy the axioms of Savage's [34] Subjective Expected Utility model supplemented by a monotone continuity condition (see Villegas, [36]). A subjective expected utility representation with a sigma-additive probability measure ensues, as stated in the next assumption.

Assumption 1: SEU Individuals. For each individual i there exists a cardinal utility function over outcomes,[5] u_i, and a unique non-atomic sigma-additive subjective probability measure over events,[6] P_i, such that the preference $\succsim^i$ is represented by the functional:

$$SEU_i(f) = E_{P_i}(u_i \cdot f) \quad , \quad \forall f \in \mathcal{F} \ .$$

[5] That is, a utility function that is unique up to an increasing linear transformation.

[6] A probability measure p is non-atomic if, for any event E with $p(E) > 0$ and any $\lambda \in (0, 1)$, there exists an event $F \subset E$ such that $p(F) = \lambda p(E)$.

A weak notion of agreement among agents is required for aggregation, wherein agents need only agree upon the strict ranking of some pair of outcomes, as stipulated next.

Assumption 2: Minimal Agreement. There are outcomes x^* and x_* such that $x^* \succ^i x_*$ for all individuals i.

The notation x^*, x_* is henceforth reserved for a specific pair of outcomes that satisfies the Minimal Agreement assumption, and all individual utilities are calibrated so as to satisfy $u_i(x^*) = 1$ and $u_i(x_*) = 0$. Next a definition of unanimously agreed-upon fair events is given.

Definition 1. An event E is a unanimously agreed-upon fair event if for every individual i, $x^*Ex_* \sim^i x_*Ex^*$.

It is supposed that social preference is represented by an MEU functional. The social MEU functional is furthermore assumed to agree with individuals on one 'fair coin toss', i.e., there exists one event unambiguously perceived as fair by all individuals and which the social planner also takes to be unambiguously fair.[7] In a companion paper (see [1]) we derive an MEU representation (along with the agreed-upon fair event) based on the assumptions of SEU individuals (Assumption 1), Minimal Agreement (Assumption 2), and a set of behavioral axioms, among them one of the two Pareto conditions that are given in the next section.

Assumption 3: MEU Social Preference. There exists a cardinal utility function, u_0 and a unique non-empty, convex, and closed set, $\mathcal{C}$ of sigma-additive probabilities,[8]

[7] We thank an anonymous referee for drawing our attention to the necessity of this assumption.

[8] We take the set $\mathcal{C}$ to be closed in the weak* topology over the space of finitely additive set functions (see a precise definition of this topology in the proofs section, p. 20). This is equivalent to the set $\mathcal{C}$ being closed under event-wise convergence, that is, if there exists a generalized sequence μ_b in $\mathcal{C}$ such that $\mu_b(A)$ converges to $\mu(A)$ for every event $A \in \Sigma$, then $\mu \in \mathcal{C}$. Consequently, $\mathcal{C}$ is compact (see Maccheroni and Marinacci [25]) and the minimum operator is well defined.

such that the relation $\succsim^0$ is represented by the functional:

$$MEU(f) = \min_{p \in \mathcal{C}} E_p(u_0 \cdot f) \quad , \quad \forall f \in \mathcal{F} \ ,$$

where for some unanimously agreed-upon fair event E, $MEU(x^* E x_*) = \frac{1}{2}u_0(x^*) + \frac{1}{2}u_0(x_*)$.

3 Aggregation results

Two Pareto criteria lay down the ground rules of how and when social preference must obey the unanimous wishes of individuals. As alluded to in the introduction, agreement among individuals on the ranking of acts under the Bayesian paradigm might be the result of double disagreement, both on tastes and on beliefs. The Pareto criteria employed in this model regard as 'legitimate' only unanimous preferences with underlying consensus on either tastes or likelihoods. This relieves the social planner from the obligation to comply with spuriously unanimous rankings that are derived from conflict.

The first Pareto criterion pertains to acts that induce the same distribution over outcomes according to all individuals. It hinges upon the definition of *socially unambiguous partitions* and *socially unambiguous acts.*

Definition 2. *A partition* $\{E_1, \ldots, E_m\}$ *is a* ***socially unambiguous partition*** *if for every individual* i*,* $x^* E_k x_* \sim^i x^* E_\ell x_*$*, for all* k *and* ℓ*.*

A ***socially unambiguous act*** *is one that is measurable with respect to a socially unambiguous partition.*

In terms of probabilities, a socially unambiguous partition $\{E_1, \ldots, E_m\}$ satisfies $P_i(E_k) = \frac{1}{m}$ for every event E_k and every probability P_i. Technically speaking, Lyapunov's Theorem guarantees that such partitions exist for every $m \in N$. A socially unambiguous act f satisfies $P_i(f = x) = P_j(f = x)$ for every outcome x and every pair of individual probabilities P_i and P_j, inducing the same distribution over outcomes according to all individual priors. Such an act is akin to a lottery with known probabilities in that at the societal level it creates no ambiguity.

Gilboa, Samet, and Schmeidler [19] introduce this next Pareto condition which applies only to socially unambiguous acts, i.e., those instances wherein the probability component of all individual priors is the same. Only tastes may vary. Whenever consensus obtains under such circumstances, this Pareto condition demands that the social planner respect it.

Lottery Pareto. For two socially unambiguous acts f and g, if $f \succ^i g$ for all individuals i, then $f \succ^0 g$.

An immediate implication of this condition is that social preference agrees with individual preferences on the strict ranking of outcomes x^* and x_*, namely $x^* \succ^0 x_*$.

Agreement on the likelihoods of events, as depicted by the Lottery Pareto criterion is likely to obtain when outcomes are generated by a well understood stochastic process. Alternatively, it may prevail when individuals form beliefs using a large body of statistical data that can be freely accessed or when well established institutions provide this information to the public.

The second Pareto condition we employ, Likelihood Pareto, is Lottery Pareto's dual (this duality is mentioned in Gilboa, Samet, and Schmeidler [19]). While Lottery Pareto applies only to acts that involve agreed-upon probabilities, Likelihood Pareto applies only to acts that return agreed-upon outcomes, x^* and x_*.

Likelihood Pareto. For two events E and F, if $x^*Ex_* \succsim^i x^*Fx_*$ for all individuals i, then $x^*Ex_* \succsim^0 x^*Fx_*$.

Likelihood Pareto states that should all individuals prefer to bet on one event over another, implying that they find the former event more likely to occur than the latter, then so does society. This condition resembles those conditions concerning aggregation of experts' opinions; in both cases, likelihoods rather than tastes are at issue (see Cres, Gilboa, and Vieille, [8]).

The main result of the paper shows that when Lottery Pareto and Likelihood

Pareto are used to aggregate SEU individual preferences into a social MEU preference then (i) the social utility is a weighted average of the individual utilities and (ii) the social set of priors is composed only of probabilities that are weighted averages of individual priors. Hence, the social planner's set of priors is contained in the convex hull of individual probabilities.

Theorem 1. *Suppose* ***SEU Individuals*** *(Assumption 1),* ***Minimal Agreement*** *(Assumption 2) and* ***MEU Social Preference*** *(Assumption 3), then Lottery Pareto and Likelihood Pareto hold if and only if the social utility is a nonnegative, nonzero combination of individual utilities and each prior in the social set of priors is a weighted average of individual priors.*

Theorem 1 characterizes social MEU preference wherein tastes (utility) and beliefs (set of priors) are both based on individual preferences – social utility is a convex combination of individual utilities and social belief is composed of convex combinations of individual beliefs. This is in line with the view that the only factors in play in the likelihood evaluations of social planners are the likelihood evaluations of individuals. Such an assumption is appropriate in situations where information employed in the formation of a social belief is already accessible to the individual members of society (and this certainly applies when the social planner is one of the members of society, so that any extra information that he or she may possess has been apprehended by at least one individual).

Note that the set of priors representing social belief is not required to contain all the individual probabilities. It is left to the social planner's discretion to determine whether to assign a low weight to some individual beliefs or even to ignore them altogether, for instance if the social planner regards some of the beliefs as too extreme or if some of the individuals are considered experts on the matter under consideration while others are not.

The full proof of Theorem 1 appears in the Appendix. It is conducted in three steps. First it is shown that the social planner's priors identify with those of individuals when all individual priors assign the same probability to an event. The proof is based upon Likelihood Pareto and on the MEU assumption which includes the existence of a fair event agreed-upon by individuals and society. After showing that the social planner's priors correspond to the individual probabilities when these agree, Lottery Pareto is invoked to derive a Harsanyi-like result, by which social utility is found to be a nonnegative combination of individual utilities. Finally, a separation theorem is used to establish that there can be no prior in the social set of priors that lies outside the convex hull of the individual probabilities, otherwise Likelihood Pareto would be contradicted.

To complement Theorem 1, the second theorem of the paper characterizes those cases where society considers all individual beliefs to be plausible and therefore includes all of them in its set of priors. Under the assumptions of Theorem 1 this is also the largest possible set of social priors. In the context of MEU models, where a larger set of probabilities corresponds to a preference that is more ambiguity averse, this specific social preference exhibits the strongest possible aversion to ambiguity.

To characterize this social preference, a condition called *Social Ambiguity Avoidance* is presumed. It states that whenever at least one individual strictly prefers to bet on an unambiguous event compared to some other event then so does society, thereby asserting that every individual likelihood judgment counts. We note that Social Ambiguity Avoidance is related to an axiom called Default to Certainty that appears in Gilboa et al. [18], which ties decision makers' objective preference to their subjective one.

Social Ambiguity Avoidance. Let F be an event and E be an unambiguous event. If $x^*Ex_* \succ^i x^*Fx_*$ for some individual i, then $x^*Ex_* \succ^0 x^*Fx_*$.

Social Ambiguity Avoidance states that failing consensus upon preferences, society opts for the alternative on which there is consensus on beliefs. That is to say, society will rank an unagreed-upon event above an agreed-upon one only when this ranking is consensual among all individuals (as is already implied by Likelihood Pareto). In any other case society favors the bet whose rewarding event is unambiguous and represents a well-understood risk. This condition is akin to the comparative notion of ambiguity aversion proposed both by Epstein [14] and by Ghirardato and Marinacci [17], wherein decision maker A is considered more ambiguity averse than decision maker B if whenever B prefers an unambiguous act, then so does A.[9] While the unambiguous acts in the two works mentioned above are either exogenous (Epstein) or constant (Ghirardato and Marinacci), the unambiguous alternatives in our model are endogenous in that they involve probabilities that are agreed upon by all members of society. It is difference in beliefs that is the source of ambiguity in society in our model. As society tends toward preference for consensual events whenever any individual member does, this implies that the social planner is more ambiguity averse than any individual member of society.

The next result shows that by adding Social Ambiguity Avoidance to the conditions of Theorem 1 social belief is restricted to the set of prior probabilities, $\mathcal{C}$ that equal the entire convex hull of individual probabilities. As a result society evaluates each act according to the minimum expected social utility over the individual priors.

Theorem 2. *Suppose* ***SEU Individuals*** *(Assumption 1),* ***Minimal Agreement*** *(Assumption 2) and* ***MEU Social Preference*** *(Assumption 3), then Lottery Pareto, Likelihood Pareto, and Social Ambiguity Avoidance hold if and only if social utility is a nonnegative, nonzero combination of individual utilities, and for any two acts f and g,*

$$f \succsim^0 g \iff \min_{i \in N} E_{P_i}(u_0 \cdot f) \geq \min_{i \in N} E_{P_i}(u_0 \cdot g) \ .$$

In comparing Theorems 1 and 2, note that the social planner's preferences under the conditions of Theorem 1 may exhibit a neutral attitude towards ambiguity in the special case where social belief reduces to a single prior probability. This possibility is excluded under the conditions of Theorem 2. When Social Ambiguity Avoidance is imposed society can no longer be ambiguity neutral (i.e. admit an SEU representation)

[9] The two works differ in their notions of ambiguity neutrality, the former assuming it is equivalent to a probability sophisticated preference and the latter assuming it is an SEU preference. However when only two outcomes are concerned, as in the axiom discussed here, the two notions coincide.

unless all individuals share a common prior. This is a manifestation of the strong form of ambiguity aversion described in Social Ambiguity Avoidance. If favoring events that are not agreed upon is allowed only under consensus, then the only way to overcome an impossibility result is to allow the social planner to exhibit a non-neutral attitude toward ambiguity.

All that is needed to prove Theorem 2 (building of proof of Theorem 1) is to establish that all individual priors must be contained in the social set of priors. This inclusion is sustained via a separation theorem, which shows that Social Ambiguity Avoidance is contradicted if individual priors reside outside the social set of priors.

3.1 Identification of tastes and beliefs

The normative appeal of our model relies on the premise that under Savage's theory tastes and beliefs of individuals are meaningful notions. This can be seen in the distinct role that each of these plays in society's decision rule and in the justifications of the two Pareto conditions that are employed in this model. A fundamental assumption that allows the identification of tastes and beliefs is that individual preferences are state-independent. Without this assumption, a unique extraction of tastes and beliefs can no longer be guaranteed. As a result the utilities and probabilities that appear in our axioms and social representation may not be faithful descriptions of the actual tastes and beliefs held by individuals.

Whether or not this assumption applies depends upon the context. In situations where the concern only involves outcomes and not their causes, as in the realized value of a stock, state-independence will commonly prevail. On the other hand, when a clear connection between payoffs and states of nature exists, as in the decision making surrounding health-related problems, this assumption may be less appropriate.

For pure state-dependent problems wherein tastes cannot be distinguished from beliefs, the impossibility results presented in the introduction do not apply, and results show that aggregation is compatible with the standard form of Pareto (see Mongin [28], Gajdos, Tallon, and Vergnaud [15], and Chambers and Hayashi [7]). When more structure is imposed upon state-dependent preferences, specifically when subjective probabilities are uniquely identified (for example in Karni, Schmeidler, and Vind [24]), aggregation may still be impossible or else be very limited (see Mongin [28]). In those cases our Pareto criteria should be modified to reflect the 'correct' probabilities and utilities under the specific state-dependent model in question.

A general criticism sometimes raised against paradigms such as ours is that both the utility function and the probability measure inherent in Savage's result are mathematical constructs devoid of any meaning. According to this view, no significance should be ascribed to these mathematical artifacts beyond the expected utility formula. This criticism, which applies to many models concerning theoretical decision-making under uncertainty, is hardly specific to our work. We thus do not discuss it in this paper. The interested reader is referred to the extensive treatment the subject receives in Gilboa, Samuelson, and Schmeidler [20] and in Gayer et al. [16].

4 Proofs

4.1 Proof of Theorem 1

Let all the individual preferences be represented by SEU functionals with utilities, $\{u_i\}_{i=1}^n$ and sigma-additive probabilities $\{P_i\}_{i=1}^n$. By the assumption of Minimal Agreement all the individual utilities can be calibrated to satisfy $u_i(x_*) = 0$ and $u_i(x^*) = 1$. Suppose that social preference $\succsim^0$ admits an MEU representation in accordance with Assumption 3 with a cardinal utility, u_0 and a nonempty, closed and convex set of sigma-additive probabilities $\mathcal{C}$ (see footnote 8). Assume that Lottery Pareto and Likelihood Pareto are satisfied. An immediate consequence is that $x^* \succ^0 x_*$.

We proceed to define mixtures of acts.

Definition 3. *Let $f = [x_1, E_1; \ldots; x_m, E_m]$ and $g = [y_1, E_1; \ldots; y_m, E_m]$ be two acts, such that $\{E_1, \ldots, E_m\}$ is a partition with respect to which both acts are measurable. For $0 \leq \alpha \leq 1$, an $\alpha : (1-\alpha)$ mixture of f and g is an act $h = [x_1, G_1; y_1, E_1 \setminus G_1; \ldots; x_m, G_m; y_m, E_m \setminus G_m]$ for events G_k that satisfy $G_k \subseteq E_k$ and $P_i(G_k) =$*

$\alpha P_i(E_k)$ for every P_i.

By Lyapunov's Theorem such events G_k exist, thus the defined mixtures must exist.

Consider the set of all events that have a unanimously agreed-upon (not necessarily rational) probability. This set of events, denoted by $\mathcal{E}$, is precisely the sigma-algebra generated by all the socially unambiguous partitions of S. All the probabilities P_i identify with each other on all socially unambiguous partitions. All these probabilities are assumed to be sigma-additive, thus they identify with each other on $\mathcal{E}$ (on account of their continuity). Denote the restriction of some (hence all) individual beliefs P_i to $\mathcal{E}$ by π.

Employing Lyapunov's Theorem again, for any real number, $\rho \in [0, 1]$ there exists an event, $E \in \mathcal{E}$ such that $\pi(E) = \rho$. The set of $\mathcal{E}$-measurable acts is henceforth referred to as the set of *lotteries*. Note that the set of lotteries is closed under the mixtures defined above.

Claim 1. *For all p in $\mathcal{C}$ and E in $\mathcal{E}$, $p(E) = \pi(E)$.*

Proof. Let E be a unanimously agreed-upon fair event for which $MEU(x^*Ex_*) = \frac{1}{2}u_0(x^*) + \frac{1}{2}u_0(x_*) = \frac{1}{2}$, as in Assumption 3. Likelihood Pareto entails that $x^*Ex_* \sim^0 x_*Ex^*$. Together with the MEU representation of $\succsim^0$ it follows that $MEU(x^*Ex_*) = \min_{p\in\mathcal{C}} p(E) = \min_{p\in\mathcal{C}} p(E^c) = 1 - \max_{p\in\mathcal{C}} p(E) = MEU(x_*Ex^*)$. Thus $\min_{p\in\mathcal{C}} p(E) = \max_{p\in\mathcal{C}} p(E) = \frac{1}{2}$, implying $p(E) = \frac{1}{2}$ for every $p \in \mathcal{C}$. For any other unanimously agreed-upon fair event F, the implied rankings $x^*Fx_* \sim^0 x_*Fx^* \sim^0 x^*Ex_*$ entail $p(F) = \frac{1}{2}$ for all $p \in \mathcal{C}$.

Now suppose a socially unambiguous partition, $\{E_1, \ldots, E_4\}$, then any union of two partition elements is a unanimously agreed-upon fair event, therefore every such union is assigned a probability $\frac{1}{2}$ according to every prior in $\mathcal{C}$. In particular, for every $p \in \mathcal{C}$, $p(E_1) + p(E_2) = p(E_1) + p(E_3) = p(E_1) + p(E_4) = 0.5$, implying $p(E_1) = \frac{1}{4}$ for every $p \in \mathcal{C}$. The same follows for every event in a four-element socially unambiguous partition. In the same manner events in every dyadic socially unambiguous partition are assigned the corresponding dyadic probability by every

$p \in \mathcal{C}$. Since all probabilities in $\mathcal{C}$ and all probabilities P_i are sigma additive by assumption, it follows that $p(E) = \pi(E)$ for every $p \in \mathcal{C}$ and every $E \in \mathcal{E}$. ■

Claim 2. *The social preference $\succsim^0$ over lotteries is represented by a vNM utility function that is a convex combination of individual utilities.*

Proof. For two socially unambiguous acts L and L' the condition in Lottery Pareto, when translated into the representation, states $E_\pi(u_i \cdot L) > E_\pi(u_i \cdot L')$ for all i. Employing the previous claim, the conclusion of the axiom translates into $E_\pi(u_0 \cdot L) > E_\pi(u_0 \cdot L')$.

Although the axiom addresses only acts that involve rationally valued π-probabilities, the same implication in representation follows for irrational probabilities as well. Suppose on the contrary that for two lotteries, L and L', which involve irrational π-probabilities, $E_\pi(u_i \cdot L) > E_\pi(u_i \cdot L')$ for all i, but $E_\pi(u_0 \cdot L') \geq E_\pi(u_0 \cdot L)$. Construct a new socially unambiguous act Q' from L' by assigning all outcomes other than the socially most favorable outcome under L' to socially unambiguous events with slightly lower probabilities than those in L', so that the new probabilities are rational, and assigning the most favorable outcome to the complementary event, which now has a higher, rational probability. Similarly construct a socially unambiguous act Q from L by amplifying the probability of the socially least favorable outcome under L, to obtain that outcomes are assigned to events with rational π-probabilities. The inequality of the social preference still holds owing to monotonicity, and for some small enough π-probabilities the strict inequalities for the individual preferences hold as well, obtaining a violation of Lottery Pareto.

The range of the vector-valued function $(E_\pi(u_0(\cdot)), E_\pi(u_1(\cdot)), \ldots, E_\pi(u_n(\cdot)))$ on the set of lotteries is convex (employing the mixtures from Definition 3), therefore according to De Meyer and Mongin ([11], by employing Minimal Agreement as well), $E_\pi(u_0(\cdot))$ is a nonnegative, non-zero linear combination of the functions $E_\pi(u_i(\cdot))$. It may be calibrated by setting $u_0(x_*) = 0$ and $u_0(x^*) = 1$, to obtain that $u_0 = \sum_{i=1}^{n} \theta_i u_i$ for non-negative weights θ_i that sum to one. ■

It is next shown that the set of probabilities entertained by the social planner, $\mathcal{C}$ is contained in $conv\{P_1, \ldots, P_n\}$, the convex hull of individual probabilities. This inclusion is proved using a separation theorem, for which further notation and definitions are required.

Denote by $B_0(S, \Sigma)$ the space of all Σ-measurable, finite-valued functions over S (equivalently, this is the vector space generated by the indicator functions of the elements of Σ), endowed with the supremum norm. Denote by $ba(\Sigma)$ the space of all bounded and finitely additive functions from Σ to $\mathbb{R}$, endowed with the total variation norm. The space $ba(\Sigma)$ is isometrically isomorphic to the conjugate space of $B_0(S, \Sigma)$. Consider an additional topology on $ba(\Sigma)$. For $\varphi \in B_0(S, \Sigma)$ and $m \in ba(S, \Sigma)$, let $\varphi(m) = \int_S \varphi dm$. Every φ defines a linear functional over $ba(\Sigma)$, and $B_0(S, \Sigma)$ is a total space of functionals on $ba(\Sigma)$.[10] The $B_0(S, \Sigma)$ topology of $ba(\Sigma)$, by definition makes a locally convex linear topological space, and the linear functionals on $ba(\Sigma)$ which are continuous in this topology are precisely the functionals defined by $\varphi \in B_0(S, \Sigma)$. This topology is called the weak* topology of $ba(\Sigma)$.

Lemma 3. *$\mathcal{C} \subseteq conv\{P_1, \ldots, P_n\}$.*

Proof. Suppose on the contrary there exists $p' \in \mathcal{C}$ such that $p' \notin conv\{P_1, \ldots, P_n\}$. By a standard separation theorem (based on the topological properties detailed above; see for instance Corollary V.2.12 in Dunford and Schwartz [13]) there exists a separating non-zero vector $\varphi = [\varphi_1, E_1; \ldots; \varphi_m, E_m] \in B_0(S, \Sigma)$ and a scalar, c such that $\varphi(q) \geq c > \varphi(p')$ for all $q \in conv\{P_1, \ldots, P_n\}$. Note that as q and p' are probabilities the separating vector φ cannot be constant.

In order to maintain all coordinates of φ between zero and one subtract $\min_k \varphi_k = \min_k \varphi_k \cdot q = \min_k \varphi_k \cdot p'$ from both sides of the two inequalities and then divide them all by the sum of the resulting coordinates (which cannot be zero as φ is non-constant). Denote the transformed vector, φ by $\alpha = [\alpha_1, E_1; \ldots; \alpha_m, E_m]$ for which $0 \leq \alpha_k \leq 1$, and not all coordinates, α_k are zero. The corresponding transformed scalar is also

[10] That is, $\varphi(m) = 0$ for every $\varphi \in B_0(S, \Sigma)$ implies that $m = 0$.

between zero and one, strictly larger than zero since $\alpha(p') \geq 0$. Denote by $\hat{c}$ the transformed scalar, c. With the new notation it holds that: $\alpha(q) \geq \hat{c} > \alpha(p')$ for all $q \in conv\{P_1, \ldots, P_n\}$.

By Lyapunov's Theorem, for every event E_k there exists an event $G_k \subseteq E_k$ such that $p'(G_k) = \alpha_k p'(E_k)$ as well as $P_i(G_k) = \alpha_k P_i(E_k)$ for all i. The act $f = [x^*, G_1; x_*, E_1 \backslash G_1; \ldots ; x^*, G_m; x_*, E_m \backslash G_m] = x^*(\cup_{k=1}^m G_k)x_*$ satisfies $E_{P_i}(u_i \cdot f) = \alpha(P_i)$ for all i, and $E_{p'}(u_0 \cdot f) = \alpha(p') \geq \min\limits_{p \in \mathcal{C}} E_p(u_0 \cdot f)$. In addition, take $F \in \mathcal{E}$ such that $\pi(F) = \hat{c}$. As $F \in \mathcal{E}$, all probabilities P_i and every $p \in \mathcal{C}$ agree that $P_i(F) = p(F) = \pi(F) = \hat{c}$, therefore $E_{P_i}(u_i \cdot (x^* F x_*)) = \min_{p \in \mathcal{C}} E_p(u_0 \cdot (x^* F x_*)) = \hat{c}$, for all i. The separation result thus yields $E_{P_i}(u_i \cdot f) \geq E_{P_i}(u_i \cdot (x^* F x_*))$, namely $f = x^*(\cup_{k=1}^m G_k)x_* \succsim^i x^* F x_*$, for all i, and at the same time, $\min\limits_{p \in \mathcal{C}} E_p(u_0 \cdot (x^* F x_*)) > \min\limits_{p \in \mathcal{C}} E_p(u_0 \cdot f)$, so that $x^* F x_* \succ^0 x^*(\cup_{k=1}^m G_k)x_*$. A contradiction to Likelihood Pareto results. It follows that $\mathcal{C} \subseteq conv\{P_1, \ldots, P_n\}$. ■

To prove the other direction of the theorem suppose that f and g are two socially unambiguous acts. Then all individual probabilities, hence all the priors in $\mathcal{C}$, agree on the distributions over outcomes induced by f and g. Each one of these acts is thus evaluated by its expected utility based on the same probabilities. If all these computations, for all utilities, u_i yield f being preferred to g, then the same will be true for any nonnegative combination of the utilities u_i, and hence for u_0. The social planner will therefore strictly prefer f to g whenever strict preference holds for all individuals, and therefore Lottery Pareto is satisfied.

Now suppose that for two acts $x^* E x_*$ and $x^* F x_*$ one is unanimously preferred over the other. Then $P_i(E) \geq P_i(F)$ for all individuals i. As each $p \in \mathcal{C}$ is a convex combination of the priors P_i it follows that $p(E) \geq p(F)$ for all $p \in \mathcal{C}$, therefore $x^* E x_* \succsim^0 x_* F x_*$ by the assumed representation of the social preference, and thus Likelihood Pareto is satisfied.

4.2 Proof of Theorem 2

Theorem 1 confirms that $\mathcal{C} \subseteq conv\{P_1, \ldots, P_n\}$. Thus for the proof of Theorem 2 the only part that must be verified is that Social Ambiguity Avoidance holds if and only if the opposite inclusion of the priors set holds, namely that the set of priors, $\mathcal{C}$ in the social preference representation contains all the individual priors.

Lemma 4. *$conv\{P_1, \ldots, P_n\} \subseteq \mathcal{C}$.*

Proof. Suppose on the contrary that there exists P_i such that $P_i \notin \mathcal{C}$. By a standard separation theorem there exists a separating non-zero vector $\varphi = [\varphi_1, E_1; \ldots; \varphi_m, E_m] \in B_0(S, \Sigma)$ and a scalar c such that $\varphi(p) \geq c > \varphi(P_i)$ for all $p \in \mathcal{C}$, where this vector cannot be constant. Once again $\min_k \varphi_k$ is subtracted from all coordinates of φ and all sides of the inequalities are divided by the sum of the (nonnegative) coordinates, so that the resulting coordinates are all between zero and one. The resulting vector is denoted by $\alpha = [\alpha_1, E_1; \ldots; \alpha_m, E_m]$, with $0 \leq \alpha_k \leq 1$ for all k, not all coordinates being zero. The transformed scalar is again between zero and one and strictly larger than zero. Let $\hat{c}$ denote a separating scalar between zero and one that is rational (Such a separating scalar exists since any separating scalar can be approached in an arbitrarily close manner by a rational number).

In the same manner as in Lemma 3 for every E_k let G_k be an event such that $G_k \subseteq E_k$ and $P_i(G_k) = \alpha_k P_i(E_k)$ for all i. As $\mathcal{C}$ is known by the previous lemma to be contained in the convex hull of the P_is it follows that $p(G_k) = \alpha_k p(E_k)$ for all $p \in \mathcal{C}$. The act $f = [x^*, G_1; x_*, E_1 \setminus G_1; \ldots; x^*, G_m; x_*, E_m \setminus G_m] = x^*(\bigcup_{k=1}^m G_k)x_*$ satisfies $E_{P_i}(u_i \cdot f) = \alpha(P_i)$ and $E_p(u_0 \cdot f) = \alpha(p)$ for all $p \in \mathcal{C}$. Furthermore, there exists a socially unambiguous event F, such that $\pi(F) = \hat{c}$, hence $E_{P_i}(u_i \cdot (x^* F x_*)) = \hat{c}$, and also $E_p(u_0 \cdot (x^* F x_*)) = \hat{c}$, for all $p \in \mathcal{C}$. The separation result thus yields $x^*(\bigcup_{k=1}^m G_k)x_* \succsim^0 x^* F x_*$, but at the same time $x^* F x_* \succ^i x^*(\bigcup_{k=1}^m G_k)x_*$, contradicting Social Ambiguity Avoidance. It follows that $conv\{P_1, \ldots, P_n\} \subseteq \mathcal{C}$. ■

It is concluded that $\mathcal{C} = conv\{P_1, \ldots, P_n\}$, and by the structure of the social set of priors it follows that social preference admits the representation, for every $f \in \mathcal{F}$,

$$MEU(f) = \min_{i \in N} E_{P_i}(u_0 \cdot f) \ .$$

In the other direction suppose that $\succsim^0$ is represented by a minimum expected utility over all individual priors. Let F be some event and let E be a socially unambiguous event. If for some individual i $P_i(E) = \pi(E) > P_i(F)$, then $\min_{j \in N} P_j(E) = \pi(E) > P_i(F) \geq \min_{j \in N} P_j(F)$, yielding $x^* E x_* \succ^0 x^* F x_*$.

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